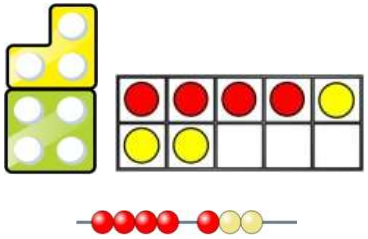
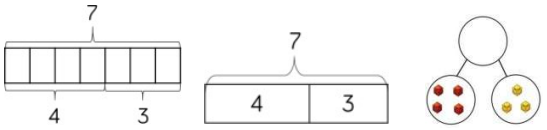
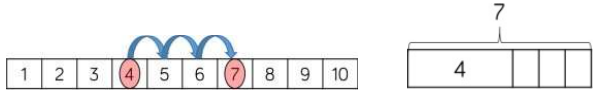
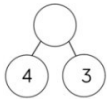
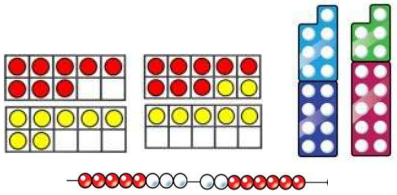
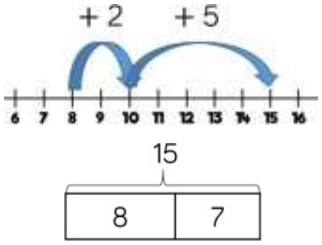
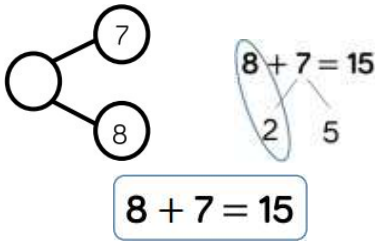
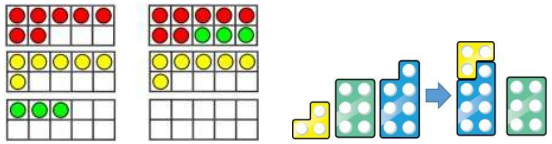
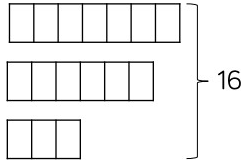
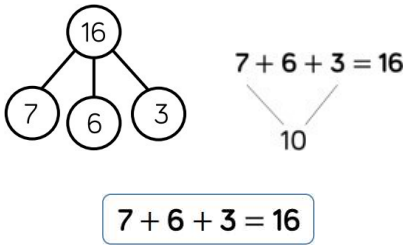
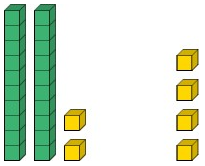
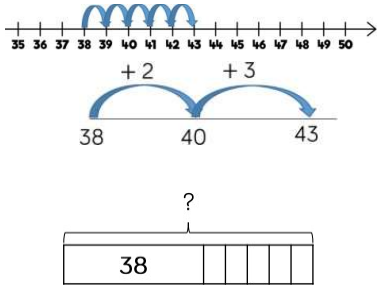
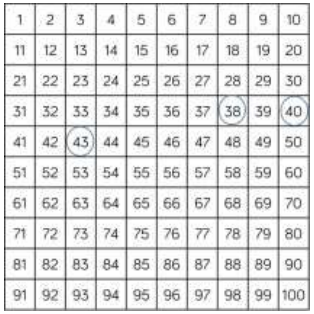
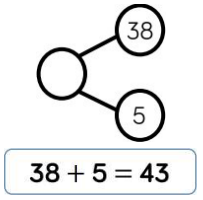
This progression sets out the strategies used at Gretton Primary School to develop understanding and proficiency in calculation skills (addition, subtraction, multiplication and division). Staff use manipulatives, pictorial representations and abstract form to develop calculation skills throughout the school and children are encouraged to use manipulatives and pictorial representations, not only to support them in calculating, but also to explain and demonstrate their understanding of mathematical concepts. Because of its progressive nature, it may be necessary to refer back to and use strategies from previous points in the progression to recap previous learning, ensure children are ready to progress or close gaps in learning.

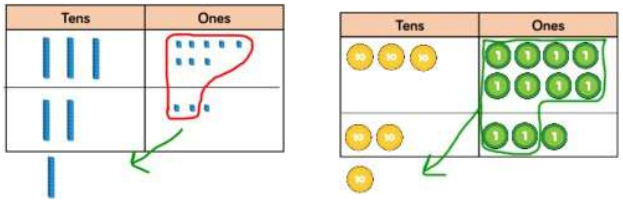
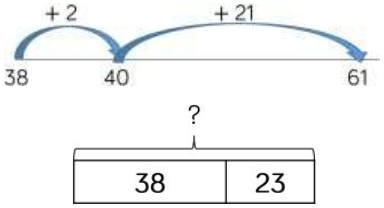
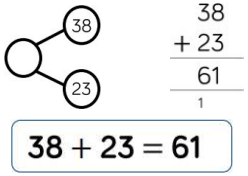
This calculation progression document aims to:

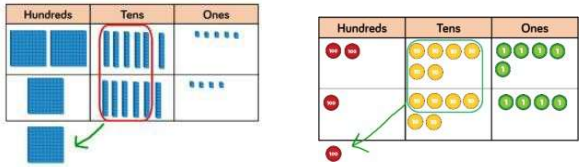
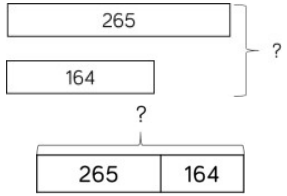
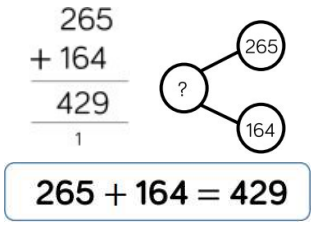
- Provide clarity on the different variations we use to teach calculation.
- Ensure consistency between classes to support children's learning.
- Help children to develop deep understanding of calculation methods.
- Support parents/carers in understanding the methods we use in school to teach the children and provide consistency when they support their children with home learning/homework.

<table>
<tr><th>Skills</th><th>Overview</th><th colspan="3">Addition
Concrete → Pictorial → Abstract</th></tr>
<tr><td>Add 1-digit numbers within 10.</td><td>Children to explore aggregation and augmentation. Augmentation is where amounts are combined. Aggregation is where an amount is added to.</td><td>  </td><td>Augmentation:
 
Aggregation:
  </td><td> 
$4 + 3 = 7$</td></tr>
</table>

Add 1 and 2-digit numbers to 20.	Children will need to learn the importance of ten ones equalling one ten. They will use different manipulatives to represent this exchange alongside pictorial representations.			

Skills	Overview	Addition Concrete → Pictorial → Abstract		
Add 3 1-digit numbers.	When adding 3 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports and develops their understanding of commutativity. Manipulative that highlight the number bonds to 10 are effective when adding 3 digit numbers.			
Adding 1-digit and 2-digit numbers to 100.	When adding single digits to a two-digit number, children should be encouraged to count on from the larger number. They should also apply their knowledge of number bonds to add more efficiently. Hundred squares and dienes can support children to find the number bond to ten.				

Add two 2-digit numbers to 100.	At this stage, encourage children to use the formal column method when calculating alongside dienes or place value counters. Children can also use a blank number line but be encouraged to make jumps of 10 to become more efficient.			

Skills	**Overview**	**Addition** **Concrete → Pictorial → Abstract**		
Add numbers with up to 3-digits.	Dienes and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure that children write out their calculation alongside any concrete resources so that they can see the links to the written column method.			

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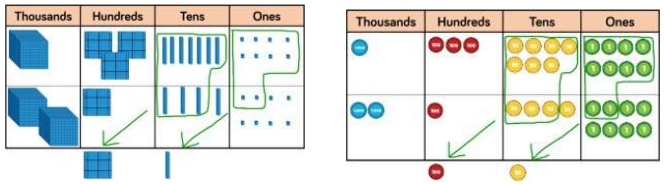
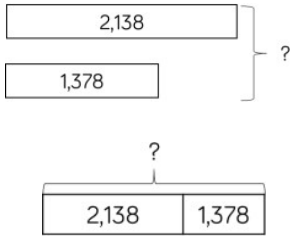
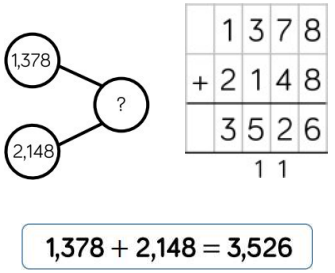
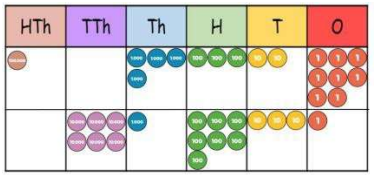
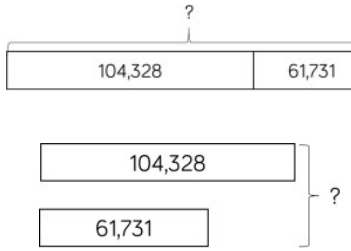
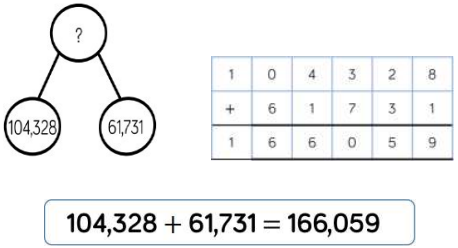
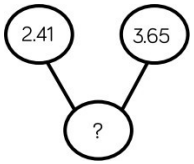
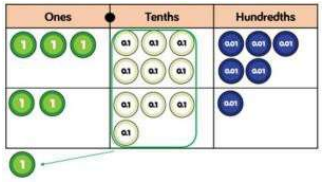
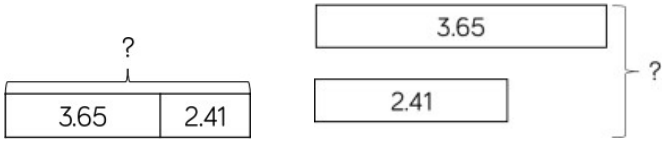
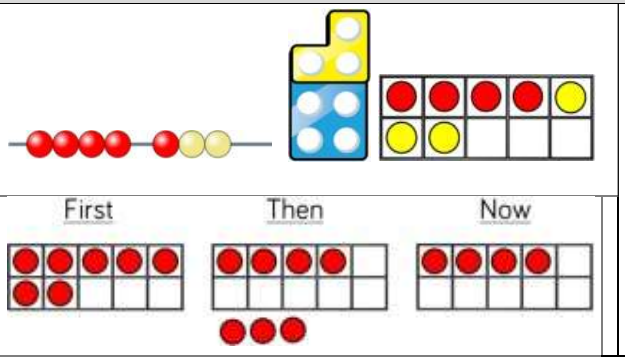
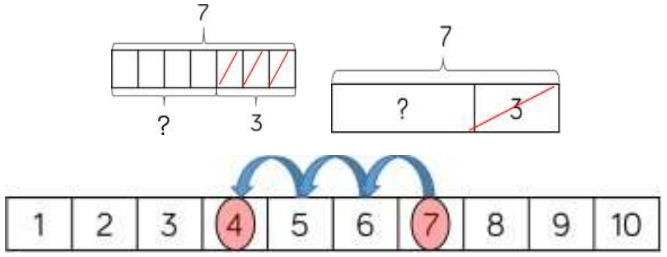
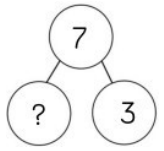
Add numbers with up to 4-digits.	Dienes and place value counters are the most effective manipulatives when adding numbers with up to 4-digits. Ensure that children write out their calculation alongside any concrete resources so that they can see the links to the written column method.			
Add numbers with more than 4-digits.	Place value counters or plain counters on a place value grid are the most effective concrete resources when adding digits of more than 4 digits. At this stage the children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.			

Skills	**Overview**	**Addition** **Concrete → Pictorial → Abstract**



Add numbers with up to 3 decimal places.	Place value counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.			$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ {\scriptstyle 1} \end{array}$ $3.65 + 2.41 = 6.06$
Add fractions.	Calculations involving fractions needs to be considered further. Currently, adapt the manipulatives and methods detailed for adding decimals and consider when converting to decimals may be appropriate for calculating.			

Skills	**Overview**	**Subtraction** **Concrete → Pictorial → Abstract**		
Subtract 1-digit numbers within 10.	Part-whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Bar models, including single and double bar models can support finding the difference.			 $7 - 3 = 4$

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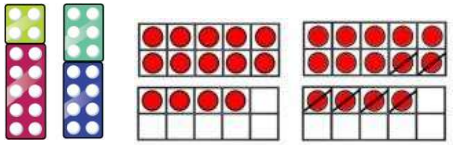
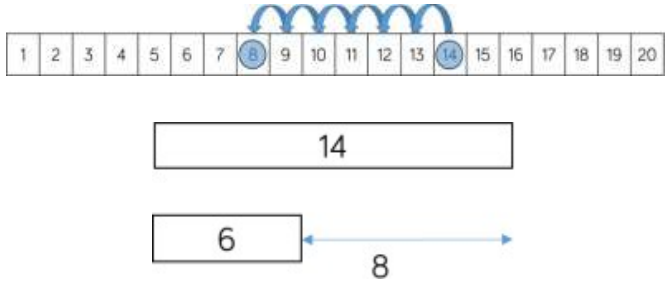
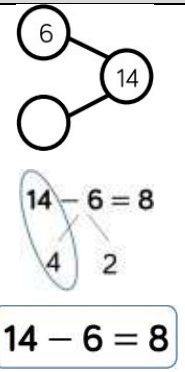
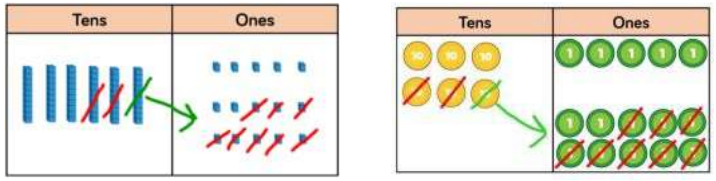
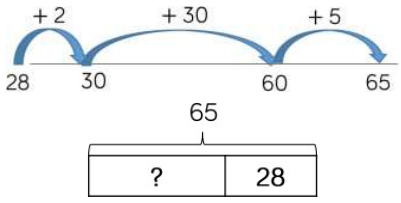
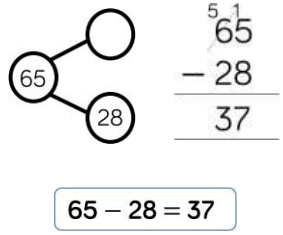
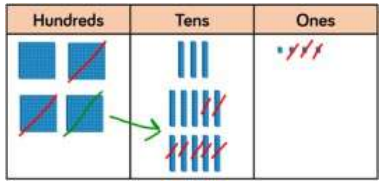
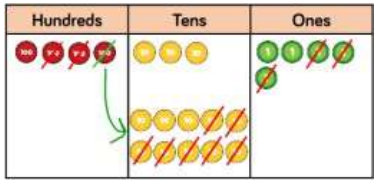
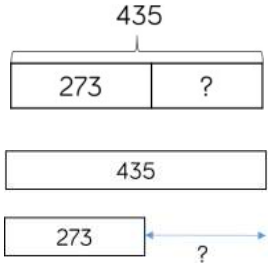
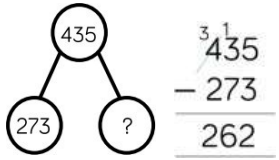
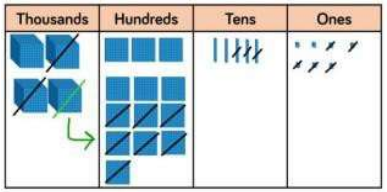
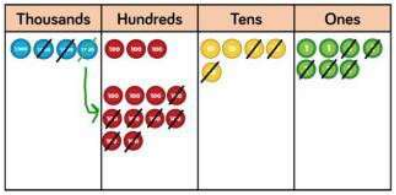
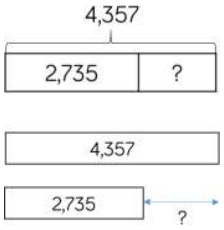
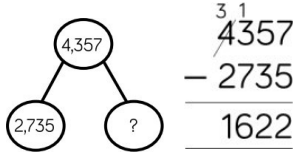
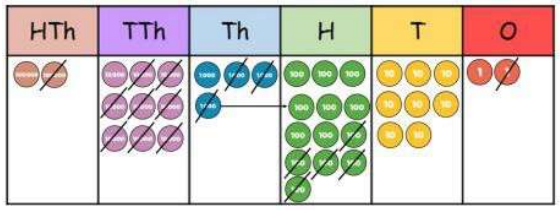
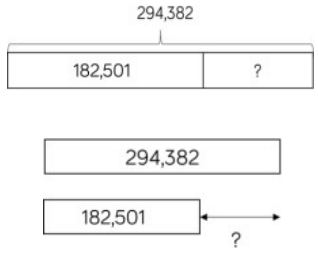
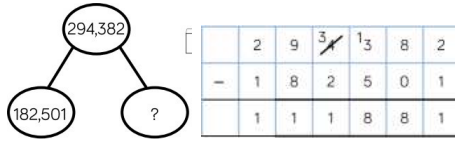
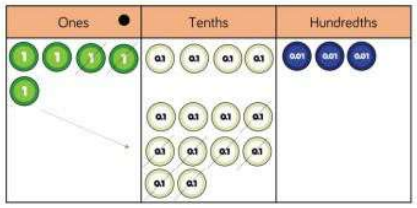
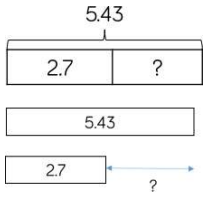
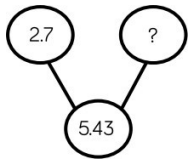
Skills	Overview	Subtraction Concrete → Pictorial → Abstract		
Subtract 1 and 2-digit numbers within 20.	When subtracting one digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Children should be encouraged to find the number bond to 10 then partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.			
Subtract 1 and 2-digit numbers to 100.	At this stage, encourage the children to use the formal column method when calculating alongside dienes or place value counters. Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.			

Subtract numbers with up to 3-digits.	Dienes and place value counters are the most effective manipulatives when subtracting numbers with up to 3-digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.	 		 **435 – 273 = 262**

Skills	**Overview**	**Subtraction** **Concrete → Pictorial → Abstract**		
Subtract numbers up to 4-digits.	Dienes and place value counters are the most effective manipulatives when subtracting numbers with up to 4-digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.	 		 **4,357 – 2,735 = 1,622**
Subtract numbers with more than 4-digits.	Place value counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4-digits. At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.			 **294,382 – 182,501 = 111,881**

Subtract numbers with up to 3 decimal places.	Place value counters on a place value grid are the most effective concrete resource when subtracting numbers with 1, 2 and 3 decimal places. Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.			 $\overset{4}{\cancel{5}}\overset{1}{.}43 - 2.7 = 2.73$ **5.43 – 2.7 = 2.73**
Subtract fraction	Calculations involving fractions needs to be considered further. Currently, adapt the manipulatives and methods detailed for subtracting decimals and consider when converting to decimals may be appropriate for calculating.			

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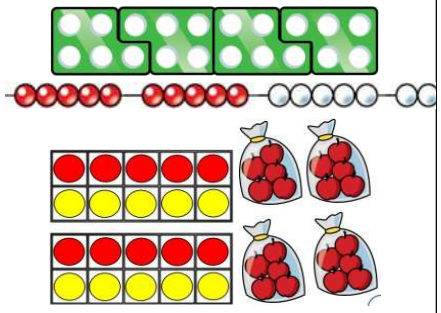
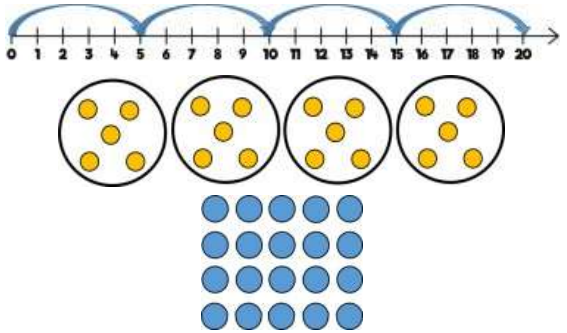
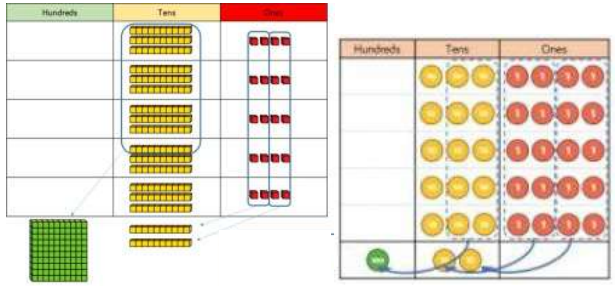
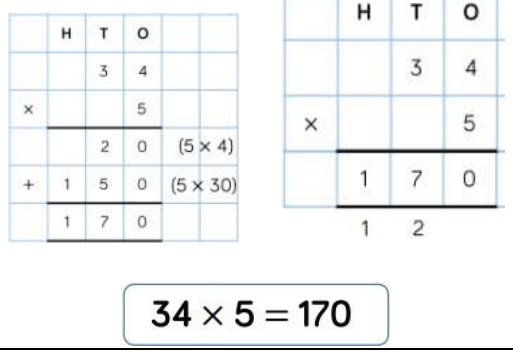
Skills	Overview	Multiplication Concrete → Pictorial → Abstract		
Solve 1 step problems using multiplication.	Children represent multiplication as repeated addition in many ways. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2, children are introduced to the multiplication symbol.			$5+5+5+5=20$ $4\times5=20$ $5\times4=20$
Multiply 2-digit numbers by 1-digit numbers.	Teachers may decide to first look at the expanded column method before moving onto the short multiplication method. The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.		Children draw counters into grids for a pictorial method.	 $34\times5=170$

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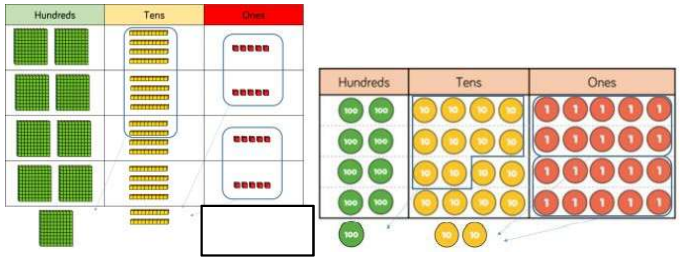
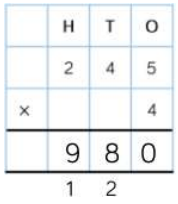
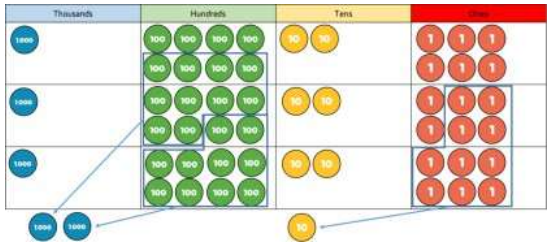
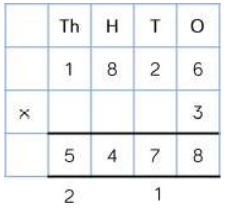
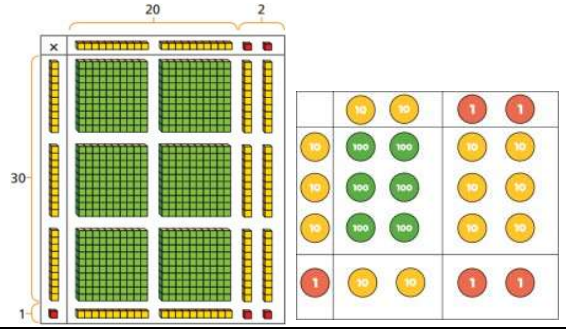
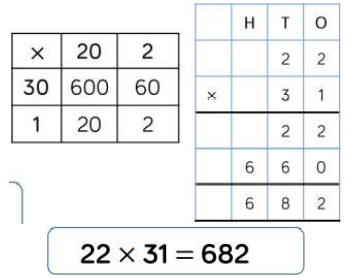
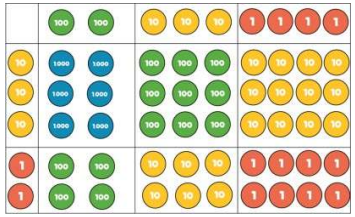
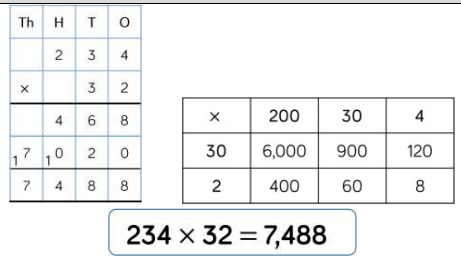
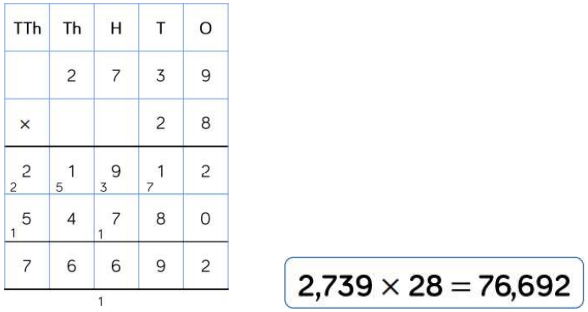
Skills	Overview	Multiplication Concrete → Pictorial → Abstract		
Multiply 3-digit numbers by 1-digit numbers.	When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short formal method. Dienes and place value counters continue to support the understanding of the written method. Limit the number of the exchanges needed in the questions and move children away from resources when multiplying larger numbers.		Children draw counters into grids for a pictorial method.	 $245 \times 4 = 980$
Multiply 4-digit numbers by 1-digit numbers.	When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.		Children draw counters into grids for a pictorial method.	 $1{,}826 \times 3 = 5{,}478$

Multiply 2-digit numbers by 2-digit numbers.	When multiplying multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by dienes. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.		Children draw counters into grids for a pictorial method.	

Skills	Overview	**Multiplication** **Concrete → Pictorial → Abstract**		
Multiply 3-digit numbers by 2-digit numbers.	Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but dienes can be used to highlight the size of the numbers. Encourage children to move towards the formal written method, seeing the links with the grid method.		Children draw counters into grids for a pictorial method.	

Multiply 4-digit numbers by 2-digit numbers.	When multiplying 4-digits by 2-digits, children should be confident in the written method. If they are still struggling with times tables, provide multiplication grids to support when they are focussing on the use of the method. Consider where exchanged digits are placed and make sure this is consistent.	

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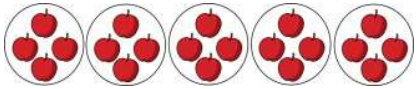
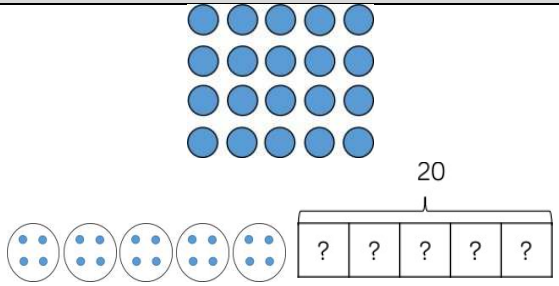
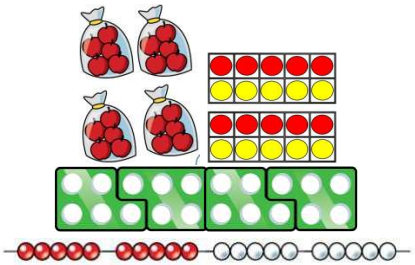
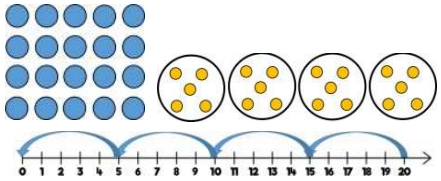
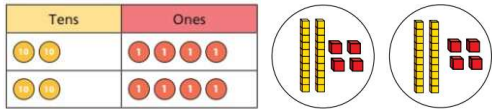
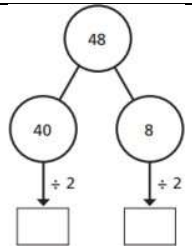
Skills	Overview	Division Concrete → Pictorial → Abstract		
Solve one-step problems using sharing.	Children solve problems by sharing into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally. In Year 2, children are introduced to the division symbol.			$20 \div 5 = 4$ There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?
Solve one step problems using grouping.	Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes, which helps to show the link between multiplication and division.			$20 \div 5 = 4$ There are 20 apples altogether. They are put in bags of 5. How many bags are there?
Divide 2-digit by 1-digit numbers with no exchange.	When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones. Dienes and place value counters can all be used to share numbers into equal groups. Part-whole models can provide children with a clear written method that matches the pictorial representation.			$48 \div 2 = 24$

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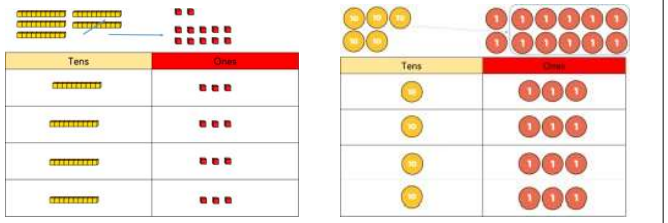
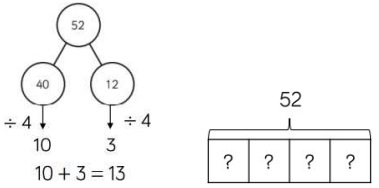
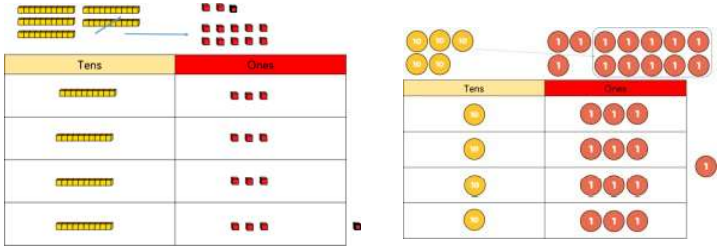
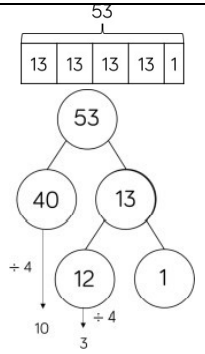
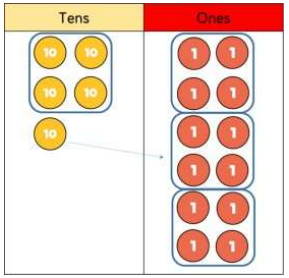
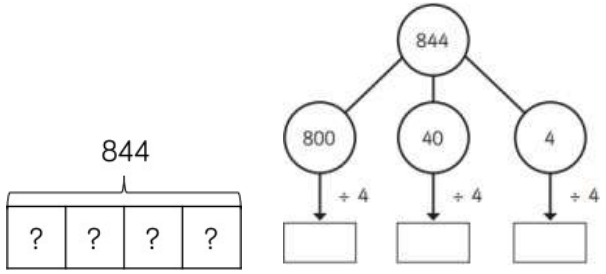
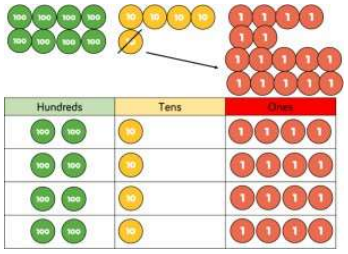
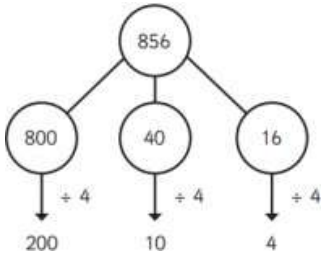
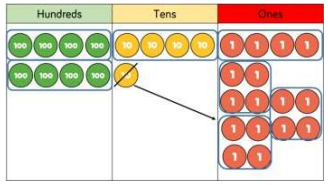
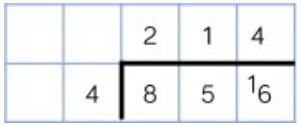
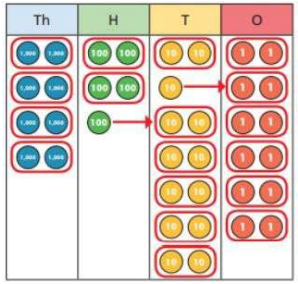
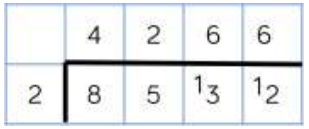
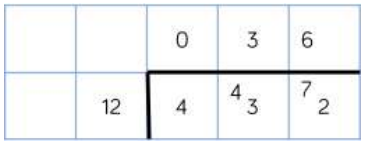
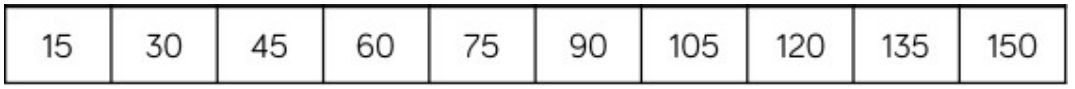
Skills	Overview	Division Concrete → Pictorial → Abstract		
Divide 2-digit by 1-digit numbers with exchanging.	When dividing numbers involving an exchange, children can use dienes and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. Flexible partitioning in the part whole model supports this method.			$52 \div 4 = 13$
Divide 2-digit by 1-digit numbers with remainders.	When dividing numbers with remainders, children can use dienes and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in part-whole models supports this method.			$53 \div 4 = 132r1$

Divide 2-digits by 1-digit numbers by grouping.	When using the short division method, children use grouping. Starting with the largest place value, they group the divisor. Language is important here. Children should consider 'How many groups of 4 tens can we make? How many groups of 4 ones can we make?' Remainders can also be seen as they are left ungrouped.		Children draw counters into grids for a pictorial method.	 52 ÷ 4 = 13

Skills	Overview	Division Concrete → Pictorial → Abstract		
Divide 3-digit by 1-digit numbers by sharing.	Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in part-whole models supports this method.			844 ÷ 4 = 211
				856 ÷ 4 = 214

Divide 3-digit by 1-digit numbers by grouping.	Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number. Place value counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group through a more pictorial method.		Children draw counters into grids for a pictorial method.	 856 ÷ 4 = 214

Skills	**Overview**	**Division** **Concrete → Pictorial → Abstract**		
Divide 4-digit by 1-digit numbers by grouping.	Place value counters can be used on a place value grid to support children to divide 4-digit numbers by 1-digit numbers. Children can also draw their own counters and group them through a pictorial method.		Children draw counters into grids for a pictorial method.	 8,532 ÷ 2 = 4,266

Divide up to 4 digit by 2-digit numbers.	When children begin to divide up to 4-digit numbers by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.	 $432 \div 12 = 36$   $7{,}335 \div 15 = 489$

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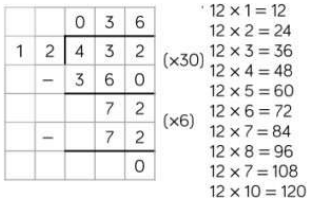
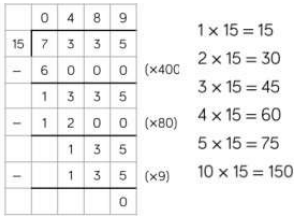
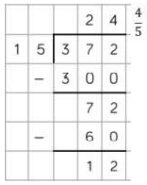
Skills	Overview	Division Concrete → Pictorial → Abstract
Divide numbers by 3-digits using long division without remainders.	Children can also divide by 2-digits using long division. Children can write out multiples to support their calculations with larger remainders. Children will solve problems with remainders where the quotient can be rounded as appropriate.	 12 × 1 = 12 12 × 2 = 24 12 × 3 = 36 12 × 4 = 48 12 × 5 = 60 12 × 6 = 72 12 × 7 = 84 12 × 8 = 96 12 × 7 = 108 12 × 10 = 120 $432 \div 12 = 36$
		$7{,}335 \div 15 = 489$  1 × 15 = 15 2 × 15 = 30 3 × 15 = 45 4 × 15 = 60 5 × 15 = 75 10 × 15 = 150
Divide numbers by 3-digits using long division with remainders.	When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question. Children can also answer the questions where the quotient needs to be rounded according to the context.	$372 \div 15 = 24\ r12$  1 × 15 = 15 2 × 15 = 30 3 × 15 = 45 4 × 15 = 60 5 × 15 = 75 10 × 15 = 150
		 $372 \div 15 = 24\frac{4}{5}$